

Reg No.: _____

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APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY
FIRST SEMESTER B.TECH DEGREE EXAMINATION(2019 SCHEME), DECEMBER 2019

Course Code: MAT101

Course Name: LINEAR ALGEBRA AND CALCULUS

(2019-Scheme)

Max. Marks: 100

Duration: 3 Hours

PART A

Answer all questions, each carries 3 marks.

- 1 Determine the rank of the matrix $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 2 & 5 \end{bmatrix}$ (3)
- 2 If 2 is an eigen value of $\begin{bmatrix} 3 & -1 & 1 \\ -1 & 5 & -1 \\ 1 & -1 & 3 \end{bmatrix}$, without using its characteristic equation, find the other eigen values. (3)
- 3 If $f(x, y) = xe^{-y} + 5y$ find the slope of $f(x, y)$ in the x-direction at (4,0). (3)
- 4 Show that $\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = 0$, where $z = e^x \sin y + e^y \cos x$ (3)
- 5 Find the mass of the square lamina with vertices (0,0) (1,0) (1,1) and (0,1) and density function $x^2 y$ (3)
- 6 Evaluate $\int_0^\infty \int_0^\infty e^{-(x^2+y^2)} dx dy$ by changing to polar coordinates. (3)
- 7 Test the convergence of the series $\sum_{k=1}^{\infty} \frac{k}{2k+1}$ (3)
- 8 Check the convergence of $\sum_{k=1}^{\infty} \frac{1}{k^{k/2}}$ (3)
- 9 Find the Taylors series for $f(x) = \cos x$ about $x = \frac{\pi}{2}$ up to third degree terms. (3)
- 10 Find the Fourier half range sine series of $f(x) = e^{-x}$ in $0 < x < 1$ (3)

PART B

Answer one full question from each module, each question carries 14 marks

Module-I

- 11 a) Solve the system of equations by Gauss elimination method. (7)

$$x + 2y + 3z = 1$$

$$2x + 3y + 2z = 2$$

$$3x + 3y + 4z = 1$$

- b) Find the eigenvalues and eigenvectors of (7)

$$\begin{bmatrix} 4 & 2 & -2 \\ 2 & 5 & 0 \\ -2 & 0 & 3 \end{bmatrix}$$

- 12 a) Find the values of λ and μ for which the system of equations (7)

$$2x + 3y + 5z = 9$$

$$7x + 3y - 2z = 8$$

$$2x + 3y + \lambda z = \mu$$

has (i) no solution (ii) a unique solution and (iii) infinite solution

- b) Find the matrix of transformation that diagonalize the matrix (7)

$$A = \begin{bmatrix} 1 & -3 & 3 \\ 3 & -5 & 3 \\ 6 & -6 & 4 \end{bmatrix}. \text{ Also write the diagonal matrix.}$$

Module-II

- 13 a) Let f be a differentiable function of three variables and suppose that (7)

$$w = f(x - y, y - z, z - x), \text{ show that } \frac{\partial w}{\partial x} + \frac{\partial w}{\partial y} + \frac{\partial w}{\partial z} = 0$$

- b) Locate all relative extrema of $f(x, y) = 4xy - y^4 - x^4$ (7)

- 14 a) Find the local linear approximation L to the function $f(x, y) = \sqrt{x^2 + y^2}$ (7)

at the point $P(3,4)$. Compare the error in approximating f by L at the point $Q(3.04, 3.98)$ with the distance PQ .

- b) The radius and height of a right circular cone are measured with errors of at (7)

most 1% and 4%, respectively. Use differentials to approximate the maximum percentage error in the calculated volume.

Module-III

- 15 a) Evaluate $\iint_R y dx dy$ where R is the region bounded by the parabolas $y^2 = 4x$ and $x^2 = 4y$. (7)
- b) Use double integral to find the area of the region enclosed between the parabola $y = \frac{x^2}{2}$ and the line $y = 2x$. (7)
- 16 a) Evaluate $\int_0^1 \int_{\frac{y}{2}}^1 e^{x^2} dx dy$ by reversing the order of integration (7)
- b) Use triple integrals to find the volume of the solid within the cylinder $x^2 + y^2 = 9$ and between the planes $z = 1$ and $x + z = 5$. (7)

Module-IV

- 17 a) Find the general term of the series $1 + \frac{1 \cdot 2}{1 \cdot 3} + \frac{1 \cdot 2 \cdot 3}{1 \cdot 3 \cdot 5} + \frac{1 \cdot 2 \cdot 3 \cdot 4}{1 \cdot 3 \cdot 5 \cdot 7} + \dots$ and use the ratio test to show that the series converges. (7)
- b) Test whether the following series is absolutely convergent or conditionally convergent $\sum_{k=1}^{\infty} \frac{(-1)^k}{\sqrt{k(k+1)}}$ (7)
- 18 a) Test the convergence of $\frac{x}{1 \cdot 2} + \frac{x^2}{2 \cdot 3} + \frac{x^3}{3 \cdot 4} + \dots + \frac{x^k}{k(k+1)} + \dots$ (7)
- b) Test the convergence of the series $\sum_{k=1}^{\infty} \frac{(k+1)!}{4! k! 4^k}$ (7)

Module-V

- 19 a) Find the Fourier series of periodic function with period 2 which is given below $f(x) = \begin{cases} -x & ; -1 \leq x \leq 0 \\ x & ; 0 \leq x \leq 1 \end{cases}$. Hence prove that $1 + \frac{1}{3^2} + \frac{1}{5^2} + \dots = \frac{\pi^2}{8}$ (7)
- b) Find the half range cosine series for $f(x) = \begin{cases} kx & 0 \leq x \leq L/2 \\ k(L-x) & L/2 \leq x \leq L \end{cases}$ (7)

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a) Find the Fourier series of $f(x) = \begin{cases} 0 & -\pi < x < 0 \\ x^2 & 0 < x < \pi \end{cases}$ (7)

b) Obtain the Fourier series expansion for $f(x) = x^2$, $-\pi < x < \pi$. (7)
