Reg No.:_____ Name:_____

APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY

SIXTH SEMESTER B.TECH DEGREE EXAMINATION, APRIL 2018

Course Code: CS302

Course Name: DESIGN AND ANALYSIS OF ALGORITHMS (CS)

Max. Marks: 100 Duration: 3 Hours

PART A

Answer all questions, each carries 3 marks. Marks

1 Is $2^{n+1} = O(2^n)$? Is $2^{2n} = O(2^n)$? Justify your answer. (3)

2 State Master's Theorem. Find the solution to the following recurrence equation using Master's theorem. (3)

a) $T(n) = 2T(n/2) + n \log n$

b) $T(n) = 2^n T(n/2) + n^n$

3 Analyse the complexity of the following program (3)

```
main ( )
{
for ( inti=1; i<=n;i=i*2)
   sum =sum+i+func(i )
}
void func(m )
{
for ( int j=1; j<=m; j++)
     Statement with O(1 ) complexity
}
```

4 State weighted rule (union by rank) and collapsing rule (path compression) applied in the disjoint set union and find operation respectively. How these rules will improve the efficiency of disjoint set operations. (3)

PART B

Answer any two full questions, each carries 9 marks.

5 a) Using iteration solve the following recurrence equation (5)

T(n)= 2 if n=1 else T(n)= 2T(n/2)+2n+3

b) Using Recursion Tree method, solve. (4)

Assume constant time for small values of n.

T(n)= 2T(n/10)+ T(9n/10)+n

6 Construct a red-black tree by inserting the keys 41, 38,31,12,19,8 into an initially empty tree. Then show the red-black trees that result from the (9)

successive deletion of the keys in the order 8,12,1, 41.

7 a) Explain the important properties of B-Tree. (2)

b) Construct a B-tree of minimum degree 3 by inserting the elements in the order given F, Q,P,K,A,L,R,M,N,X,Y,D,Z,E,H,T,V,W,C. from the constructed tree delete A,P,Q,R,T. (7)

PART C

Answer all questions, each carries 3 marks.

8 List and explain the characteristic properties associated with a problem that can be solved using dynamic programming. (3)

9 Let G be a weighted undirected graph with distinct positive edge weights. If every edge weight is increased by same value, will the minimum cost spanning tree and shortest path between any pair of vertices change. Justify your answer. (3)

10 Consider a complete undirected graph with vertex set {0, 1, 2, 3, 4}. Entry Wij in the matrix W below is the weight of the edge {i, j}. What is the minimum possible weight of a spanning tree T in this graph such that vertex 0 is a leaf node in the tree T? (3)

$$W = \begin{pmatrix} 0 & 1 & 8 & 1 & 4 \\ 1 & 0 & 12 & 4 & 9 \\ 8 & 12 & 0 & 7 & 3 \\ 1 & 4 & 7 & 0 & 2 \\ 4 & 9 & 3 & 2 & 0 \end{pmatrix}$$

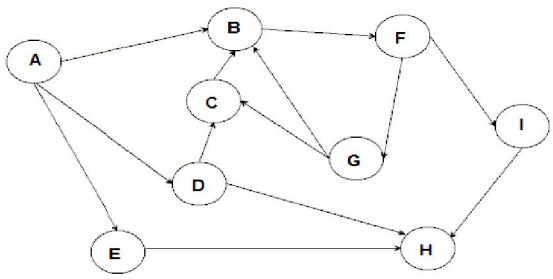
11 Let (u,v) be a minimum-weight edge in a graph G. Show that (u,v) belongs to some minimum spanning tree of G. (3)

PART D

Answer any two full questions, each carries 9 marks.

12 a) Write down DFS algorithm and analyse the time complexity. What are different classification of edges that can be encountered during DFS operation and how it is classified? (4)

b) (5)



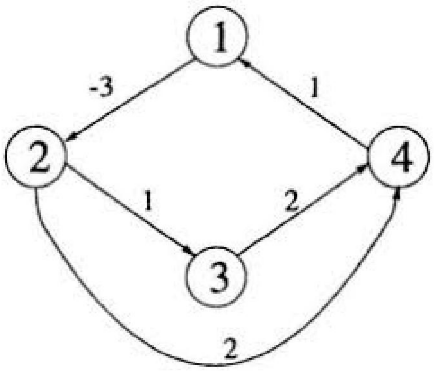
Perform DFS traversal on the above graph starting from node A. Where multiple node choices may be available for next travel, choose the next node in alphabetical order. Classify the edges of the graph into different category.

13 a) Write and explain an algorithm to find the optimal parenthesization of matrix chain product whose sequence of dimension is given. (5)

b) Write and explain merge sort algorithm using divide and conquer strategy. Also analyse the complexity. (4)

14 a) Write down and explain Bellman Ford algorithm. Will your algorithm detect all negative cycles in the graph. Justify your answer. (5)

b) Apply Bellman Ford algorithm on the graph given below. Assume Node 1 as source vertex. (4)

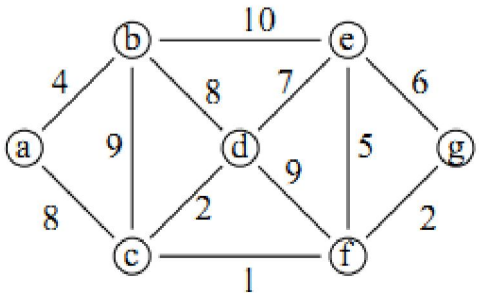


PART E

Answer any four full questions, each carries 10 marks.

15 a) Write down Prim's algorithm and analyse the complexity. (4)

b) Apply Prim's algorithm on the graph given below. (6)



16 a) Consider the following algorithm to determine whether or not an undirected graph has a clique of size k. First, generate all subsets of the vertices containing exactly k vertices. Next, check whether any of the sub-graphs induced by these subsets is complete (i.e. forms a clique). 4

Why is this not a polynomial-time algorithm for the clique problem, thereby

implying that P = NP?

b) Prove that CLIQUE problem is NP-complete.

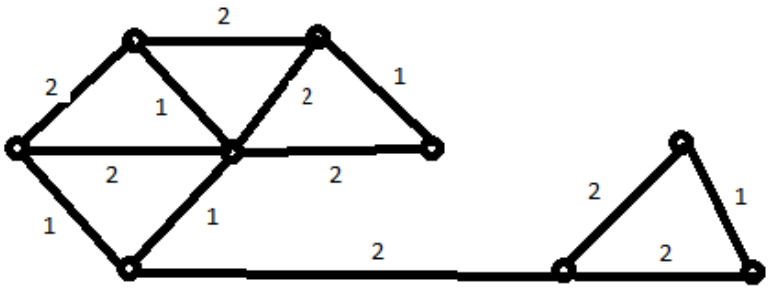
17 Explain the concept of Backtracking. Explain how 4 Queen problem can be solved using backtracking. Draw the state space tree corresponding to 4 Queen problem. (10)

18 Define Travelling Salesman Problem (TSP). Explain the basic steps that are to be followed to solve TSP using branch and bound. Illustrate with an example. (10)

19 a) State fractional knapsack problem. Give an algorithm for fractional knapsack problem using greedy strategy. (5)

b) Find an optimal solution to the fractional knapsack problem for an instance with number of items 7, Capacity of the sack W=15, profit associated with the items (p1,p2,…,p7)= (10,5,15,7,6,18,3) and weight associated with each item (w1,w2,…,w7)= (2,3,5,7,1,4,1). (4)

20 a) Find the number of distinct minimum spanning trees for the weighted graph below (4)



b) Consider a weighted complete graph G on the vertex set {v1,v2,…,vn} such that the weight of the edge (vi,vj) is 2|i-j|. Find the weight of a minimum spanning tree of G. (3)

c) Specify the difference between divide and conquer strategy and dynamic programming. (3)

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