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APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY

Fifth Semester B.Tech Degree (S,FE) Examination January 2022 (2015 Scheme)

Course Code: CS309

Course Name: GRAPH THEORY AND COMBINATORICS

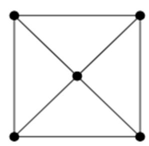
Max. Marks: 100 Duration: 3 Hours

PART A

Answer all questions, each carries 3 marks. Marks

1 Assume a graph G has n number of vertices (n>4) and it's complement graph G′ is the same. Find the minimum possible value of n. Justify your answer. (3)

2 State with valid reasons whether the given graph is Euler or not. (3)



3 Prove the statement, "If a graph (connected or disconnected) has exactly two vertices of odd degree, then there must be a path joining these two vertices". (3)

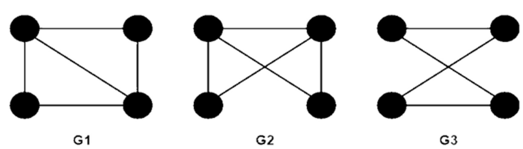
4 Construct separate digraphs for representing symmetric, transitive and equivalence relations. (3)

PART B

Answer any two full questions, each carries 9 marks.

5 a) Define complete graph. Does a complete graph contain Hamiltonian circuit? Consider a complete graph with 7 vertices, how many edge disjoint Hamiltonian circuits it has? (3)

b) Of the given graphs, determine which of them are isomorphic graphs? (6)

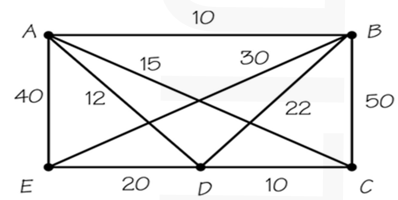


6 a) Prove the theorem, 'A simple graph with n vertices and k components can have at-most (n-k)(n-k+1)/2 edges. (4)

b) An ordered n-tuple (d_1, d_2,....,d_n) with d_1 >= d_2>=... >= d_n is called graphic if there exists a simple undirected graph with n vertices having degrees d_1, d_2, ..., d_n respectively. Which of the following is/are graphic? (5)

I. (5,5,5,5,5,5,5,5), II.(4,4,4,3,2,2,1), III.(4,4,3,3,3,2,2,2), IV.(3,2,2,1,1,1)

7	a)	State travelling salesman problem. Consider a weighted graph as below. Find and draw the minimum cost travelling salesman's tour for it. Also mention the cost.	(5)



	b)	Define the terms: (i) Simple Graph (ii) Finite Graph (iii) Infinite Graph (iv) Null Graph.	(4)

PART C

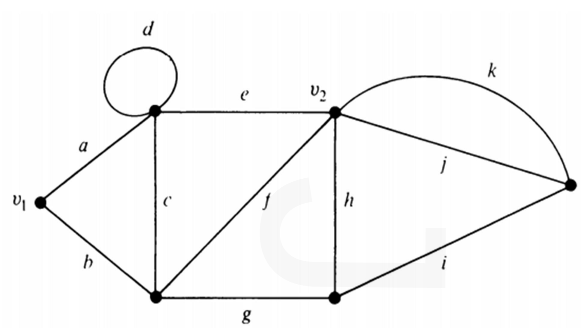
Answer all questions, each carries 3 marks.

8		Define the terms: (i) Vertex Connectivity (ii) Cut Vertex (iii) Separable Graph	(3)
9		If G is a planar graph, then any plane drawing of G divides the plane into regions, called faces. One of these faces is unbounded, and is called the infinite face. If f is any face, then the degree of f is the number of edges encountered in a walk around the boundary of the face f. If all faces have the same degree say g, then G is face-regular of degree g. Consider a graph with face regular degree of 5 and 8 vertices, then find the number of edges in the graph.	(3)
10		Prove that "Every cut set in a connected graph G must contain at least one branch of every spanning tree of G "	(3)
11		State the different metric properties of distance.	(3)

PART D

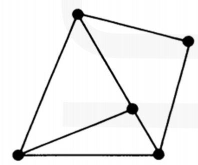
Answer any two full questions, each carries 9 marks.

12	a)	Define spanning tree. Find and draw two different spanning trees from the graph given below:	(3)



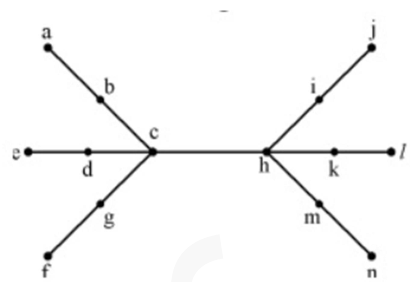
	b)	For the given graph below, find any one spanning tree contained in it and determine the fundamental cut-sets associated with that spanning tree. Then verify the theorem "With respect to a given spanning tree T, a branch *b* that	(6)

determines a fundamental cut-set S is contained in every fundamental circuit associated with the chords in S".

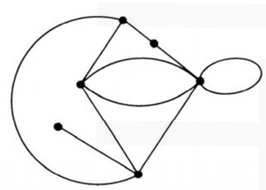


13 a) With proper arguments and facts prove the statement, "The edge connectivity of a graph cannot exceed the degree of the vertex with the smallest degree in G. " (3)

b) Find the centre, radius and diameter of the tree given below: (6)



14 a) Find the geometric dual for the given graph. (4)



b) How many labelled trees are possible with 4 vertices? Draw eight different labelled trees with 4 vertices A, B, C and D. (5)

PART E

Answer any four full questions, each carries 10 marks.

15 a) With an example compare the Edge listing and Two Linear Arrays form of computer representation for graphs. (4)

b) With a neat flow chart explain the algorithm for determining the connectedness and components for a graph. (6)

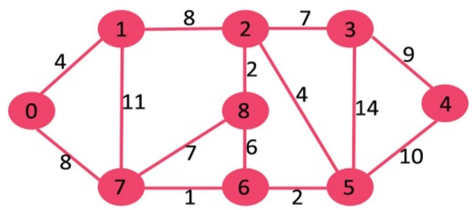
16 a) State the different properties of an incidence matrix representation of a graph. (4)

b) Given below are the adjacency matrix representations of two graphs. Draw the graph corresponding to each matrix. (Note: Assume suitable vertex name if not given). (6)

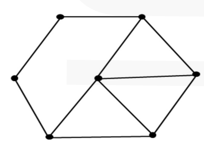
(i)
$$\begin{array}{c|cccccc} & v_1 & v_2 & v_3 & v_4 & v_5 & v_6 \\ \hline v_1 & 0 & 1 & 0 & 0 & 1 & 1 \\ v_2 & 1 & 0 & 0 & 1 & 1 & 0 \\ v_3 & 0 & 0 & 0 & 1 & 0 & 0 \\ v_4 & 0 & 1 & 1 & 0 & 1 & 1 \\ v_5 & 1 & 1 & 0 & 1 & 0 & 0 \\ v_6 & 1 & 0 & 0 & 1 & 0 & 0 \end{array}$$

(ii)
$$\begin{pmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{pmatrix}$$

17 Apply Dijkstra's algorithm to find shortest path in the given graph starting with vertex '0' as source. (10)



18 a) Find at-least 6 circuits for the given graph and generate the corresponding circuit matrix representation with the circuits obtained. (Note: Assume suitable names for the vertices and edges.) (7)



b) State the different properties of a path matrix representation of a graph. (3)

19 a Prove that the rank of an incidence matrix of a connected graph with n vertices is n-1. (4)

b Describe the steps invloved in the Prim's algorithm for computing the minimum spanning tree of a given graph. (6)

20 a) Prove the statement, "If B_f is a fundamental circuit matrix of a connected graph G with e edges and n vertices, rank of $B_f = e - n + 1$." (4)

b) With an example state how a cut-set matrix of a graph is generated. Also state the different properties of the cut-set matrix representation. (6)

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