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Name: _____

APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY
V SEMESTER B.TECH DEGREE EXAMINATION, DECEMBER 2018

Course Code: CS309

Course Name: GRAPH THEORY AND COMBINATORICS

Max. Marks: 100

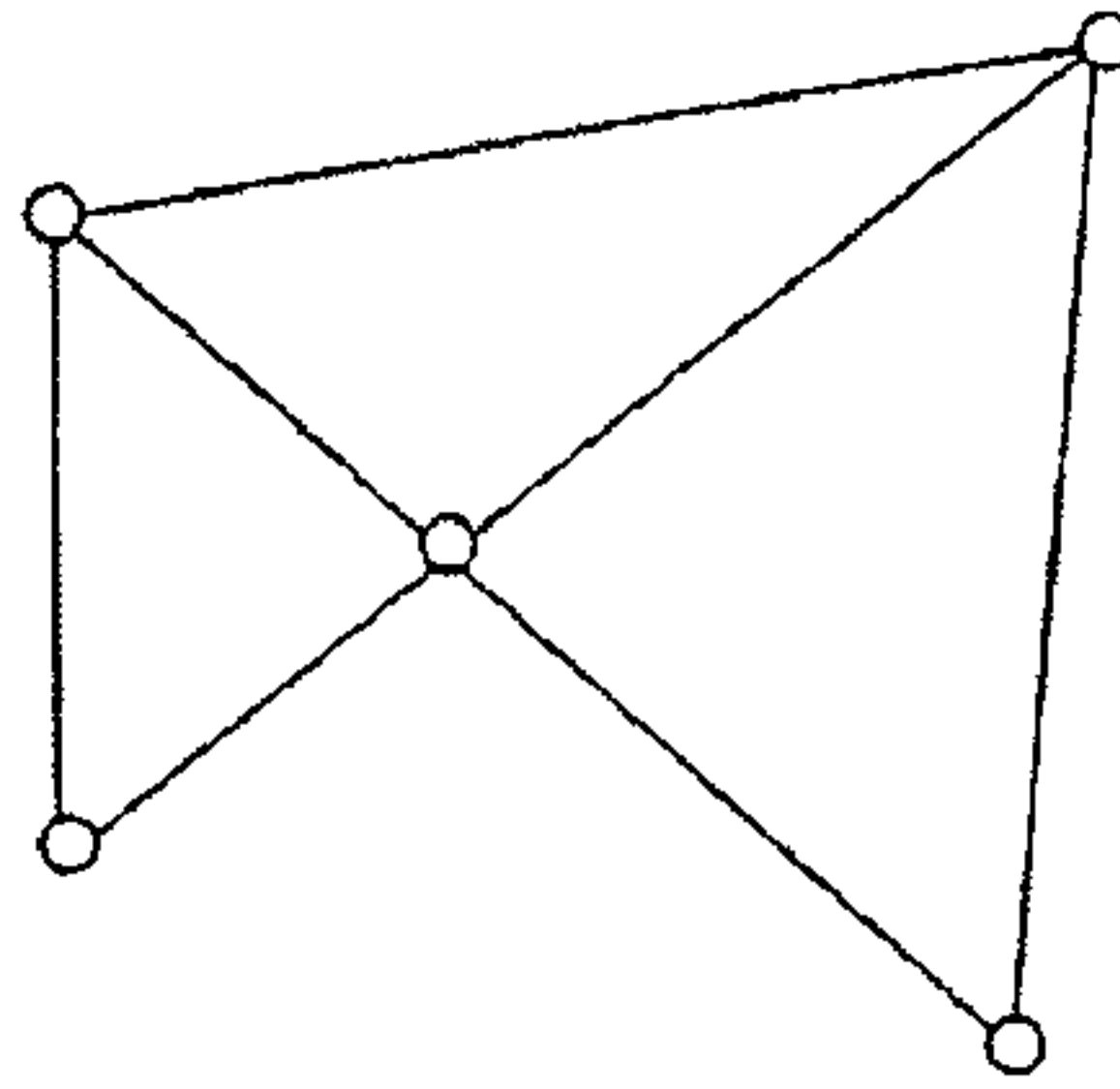
Duration: 3 Hours

PART A

Answer all questions, each carries 3 marks.

Marks

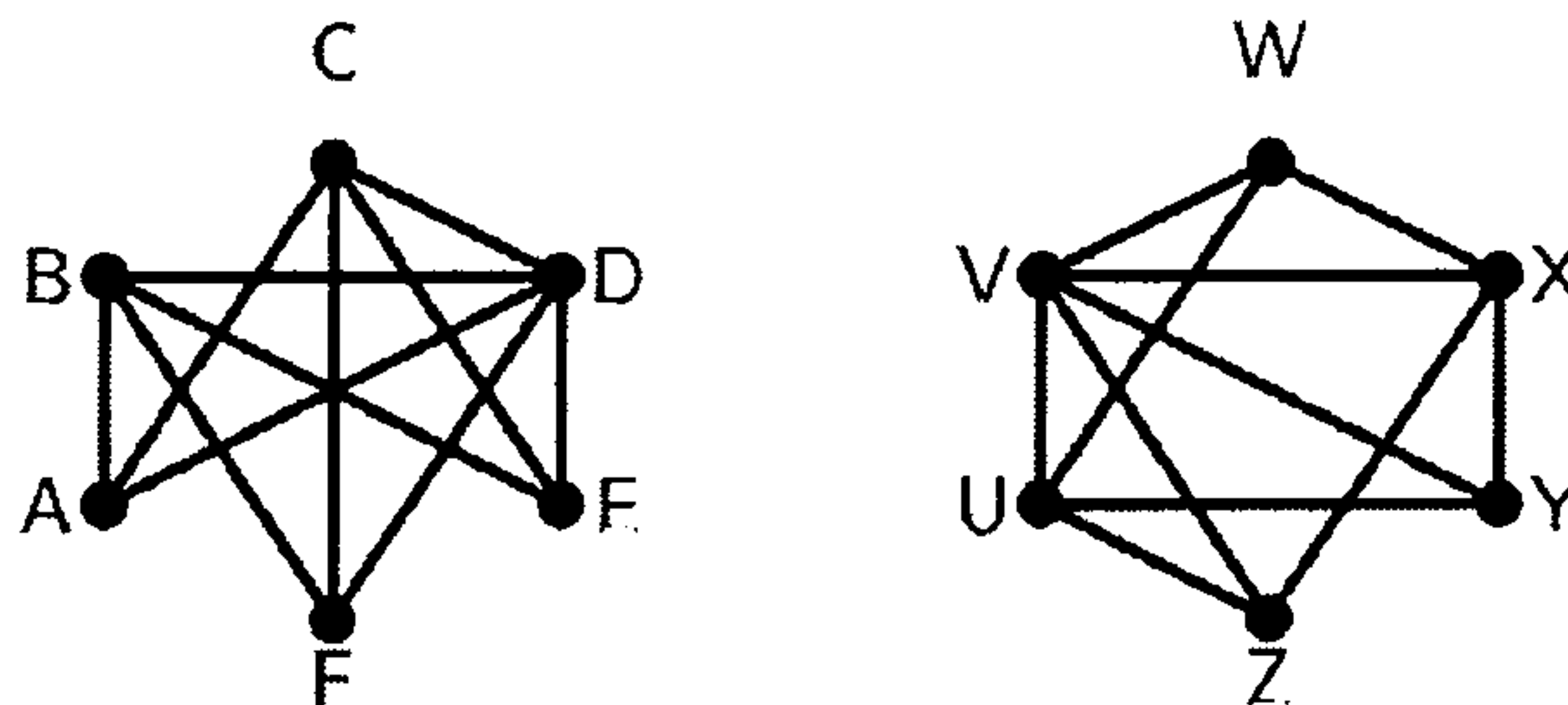
- 1 Prove that the number of vertices of odd degree in a graph is always even (3)
- 2 Show that in a simple graph with n vertices, the maximum number of edges is $n(n-1)/2$ and the maximum degree of any vertex is $n-1$. (3)
- 3 Differentiate between complete symmetric and complete asymmetric graph with an example each. (3)
- 4 State Dirac's Theorem and check its applicability in the following graph, G (3)



PART B

Answer any two full questions, each carries 9 marks.

- 5 a) Define isomorphism between graphs? Are the two graphs below isomorphic? Justify (5)



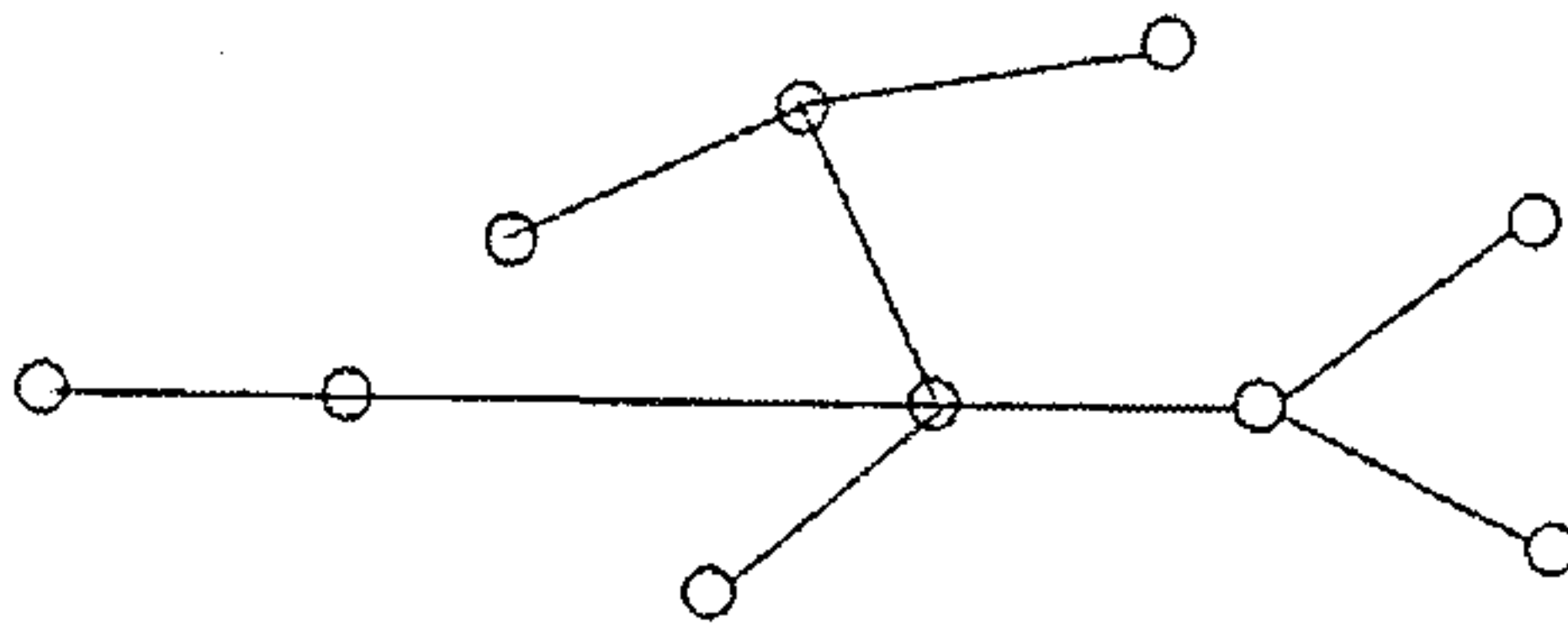
- b) Consider a complete graph G with 11 vertices. (4)
 1. Find the maximum number of edges possible in G .
 2. Find the number of edge-disjoint Hamiltonian circuits in G .
- 6 a) A connected graph G is an Euler graph if and only if all vertices of G are of even degree. Prove the statement. (6)
- b) There are 37 telephones in the city of Istanbul, Turkey. Is it possible to connect them with wires so that each telephone is connected with exactly 7 others? Substantiate your answer with graph concepts. (3)
- 7 a) Give any two applications of graphs. Explain. (2)

- b) Define Hamiltonian circuit. Give an example. What general class of graphs is guaranteed to have a Hamiltonian circuit? Also draw a graph that has a Hamiltonian path but not a Hamiltonian circuit. (4)
- c) Prove that if a connected graph G is decomposed into two subgraphs g_1 and g_2 , there must be at least one vertex common between g_1 and g_2 . (3)

PART C

Answer all questions, each carries 3 marks.

- 8 Prove that the distance between vertices of a connected graph is a metric. (3)
- 9 i) Find the eccentricity of all vertices in G given below and also mark the center of G . (3)

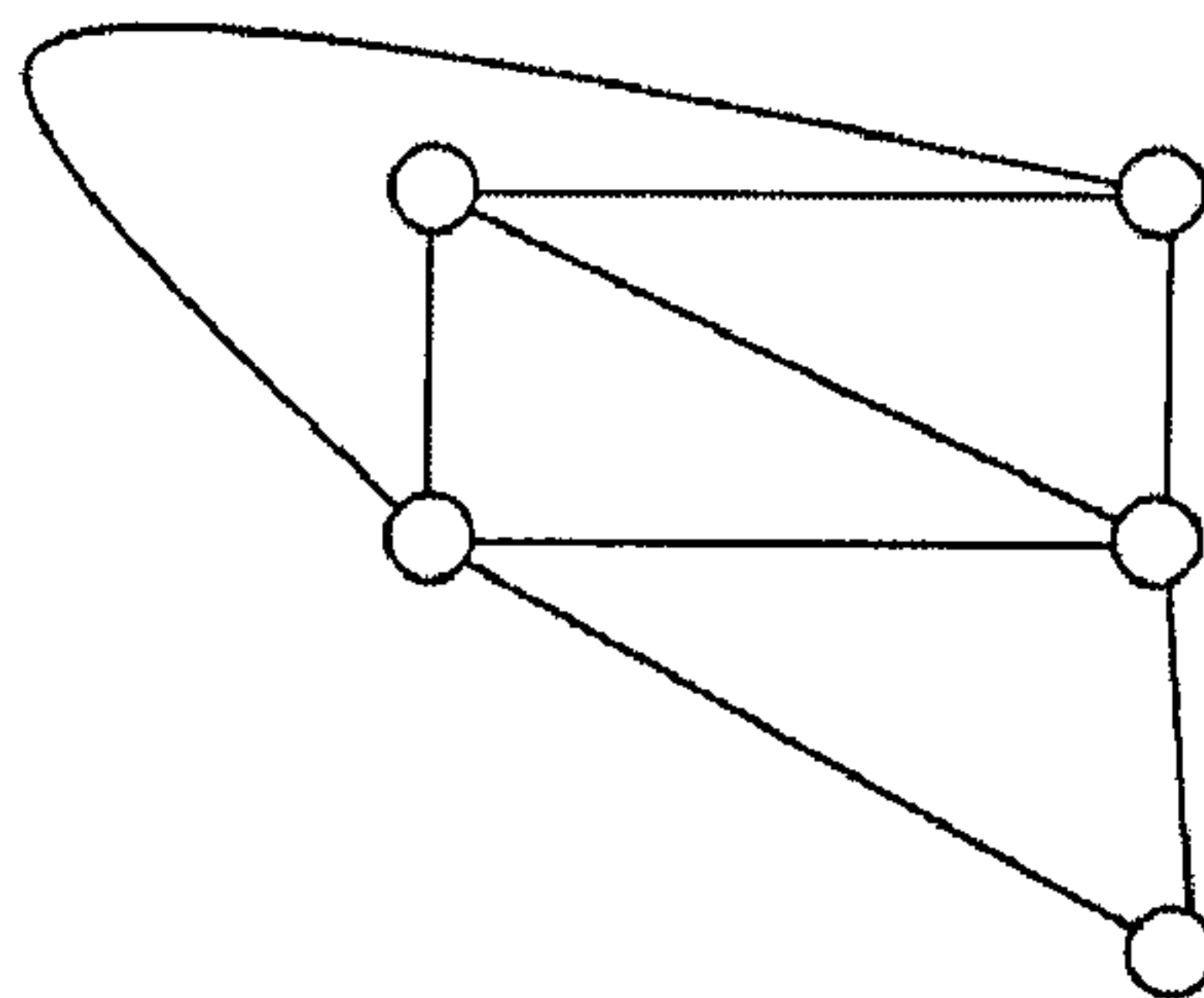


- ii) Find the number of possible labelled trees that can be constructed with 50 vertices. (3)
- 10 Draw the two simplest non-planar graphs and also mention their properties. (3)
- 11 What is the necessary and sufficient condition for two graphs to be duals of each other? Prove. (3)

PART D

Answer any two full questions, each carries 9 marks.

- 12 a) Draw the geometric dual (G^*) of G given and also write about the relationship between a planar graph G and its dual G^* . (6)



- b) Define rooted binary tree with an example. (3)
- 13 a) Find the number of edges and vertices of a graph G if its rank and nullity are 6 and 8 respectively. (2)
- b) Prove the statement, "Every circuit has an even number of edges in common with any cut-set". (4)
- c) Consider a binary tree with four weighted pendant vertices. Let their weights be 0.5, 0.12, 0.3 and 0.11. Construct a binary tree with minimum weighted path length. (3)
- 14 a) Define cut sets with an example. Give an application of finding cut-sets or edge (4)

connectivity

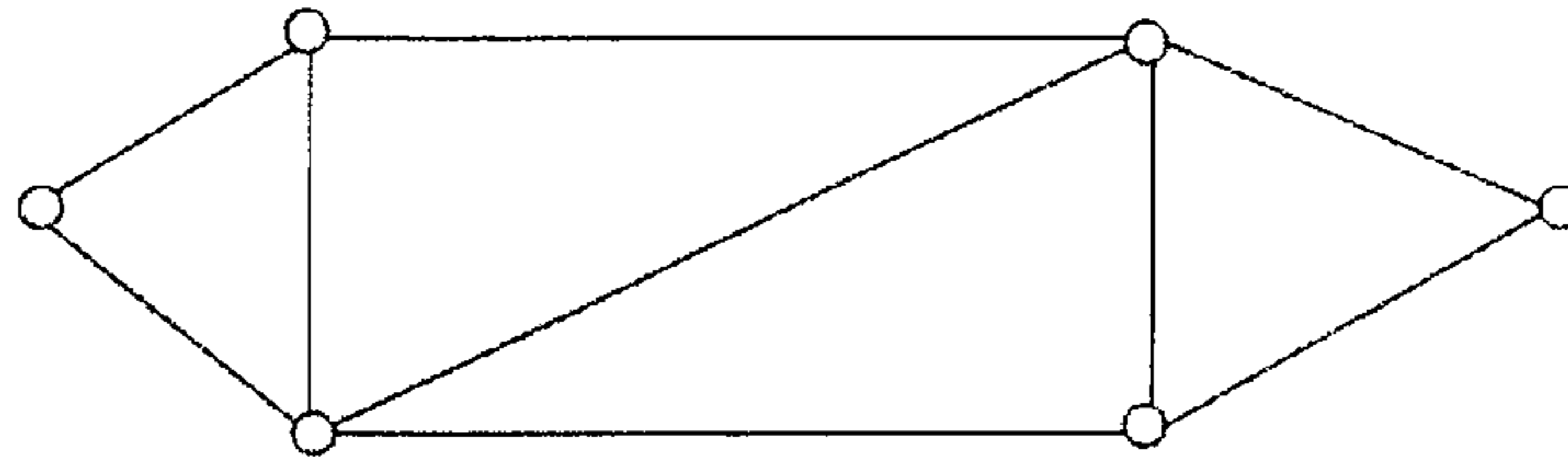
- b) Define spanning tree. Show that the edges forming a spanning tree in a planar graph G correspond to the edges forming a set of chords in the dual G^* (5)

PART E

Answer any four full questions, each carries 10 marks.

- 15 a) Draw the flow chart of spanning tree algorithm and also clearly mark the five conditions to be tested in connection with the spanning tree construction in the flowchart (6)

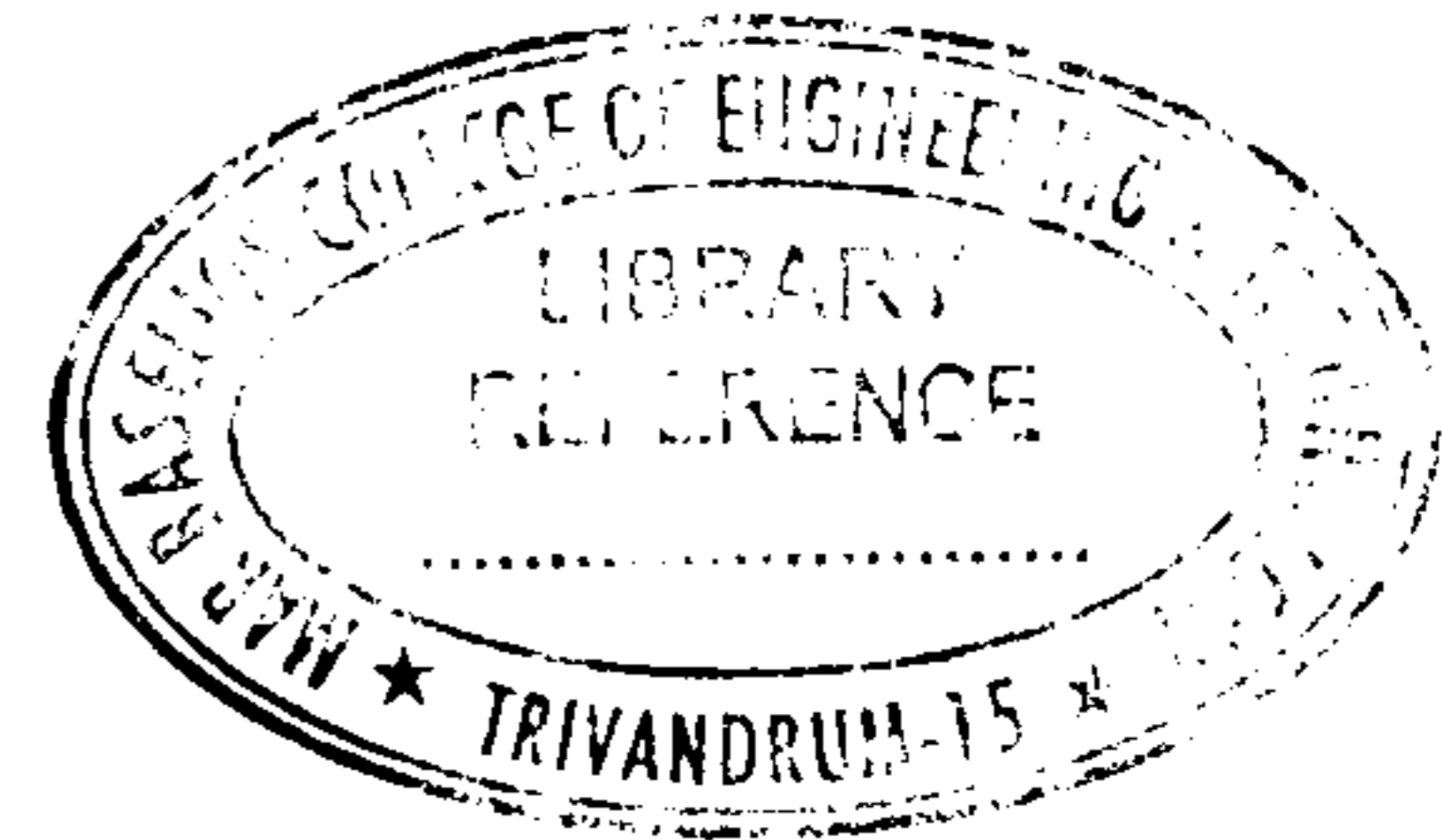
- b) Obtain a cut-set matrix for the following graph: (4)



- 16 a) Draw the flowchart to determine the components of a graph. (6)

- b) Define adjacency matrix and construct a graph from the following adjacency matrix: (4)

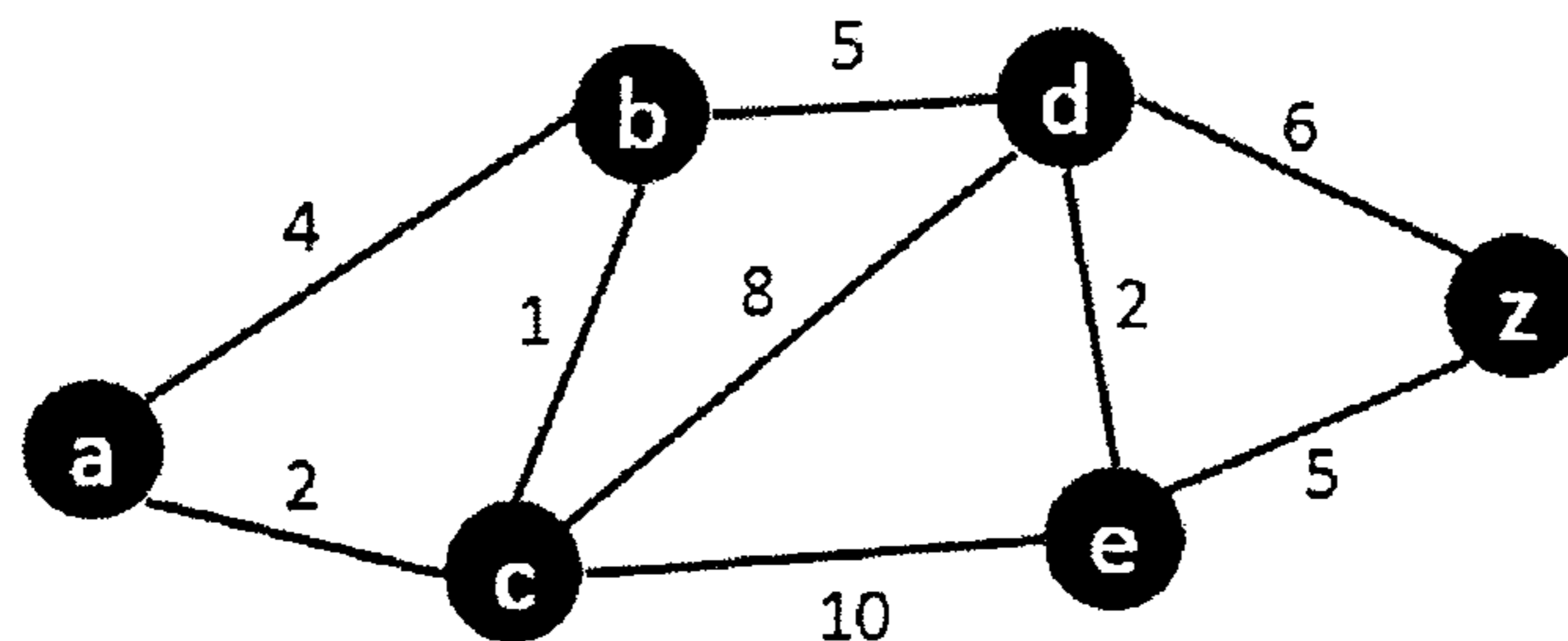
$$\begin{pmatrix} 0 & 1 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 \end{pmatrix}$$



- 17 a) Write edge listing and successor listing methods used in computer representation of graphs. (4)

- b) Two graphs G_1 and G_2 are isomorphic if and only if their incidence matrices $A(G_1)$ and $A(G_2)$ differ only by permutations of rows and columns (6)

- 18 a) Write the Dijkstra's Shortest Path Algorithm and apply this algorithm to find the shortest path between a and z (6)

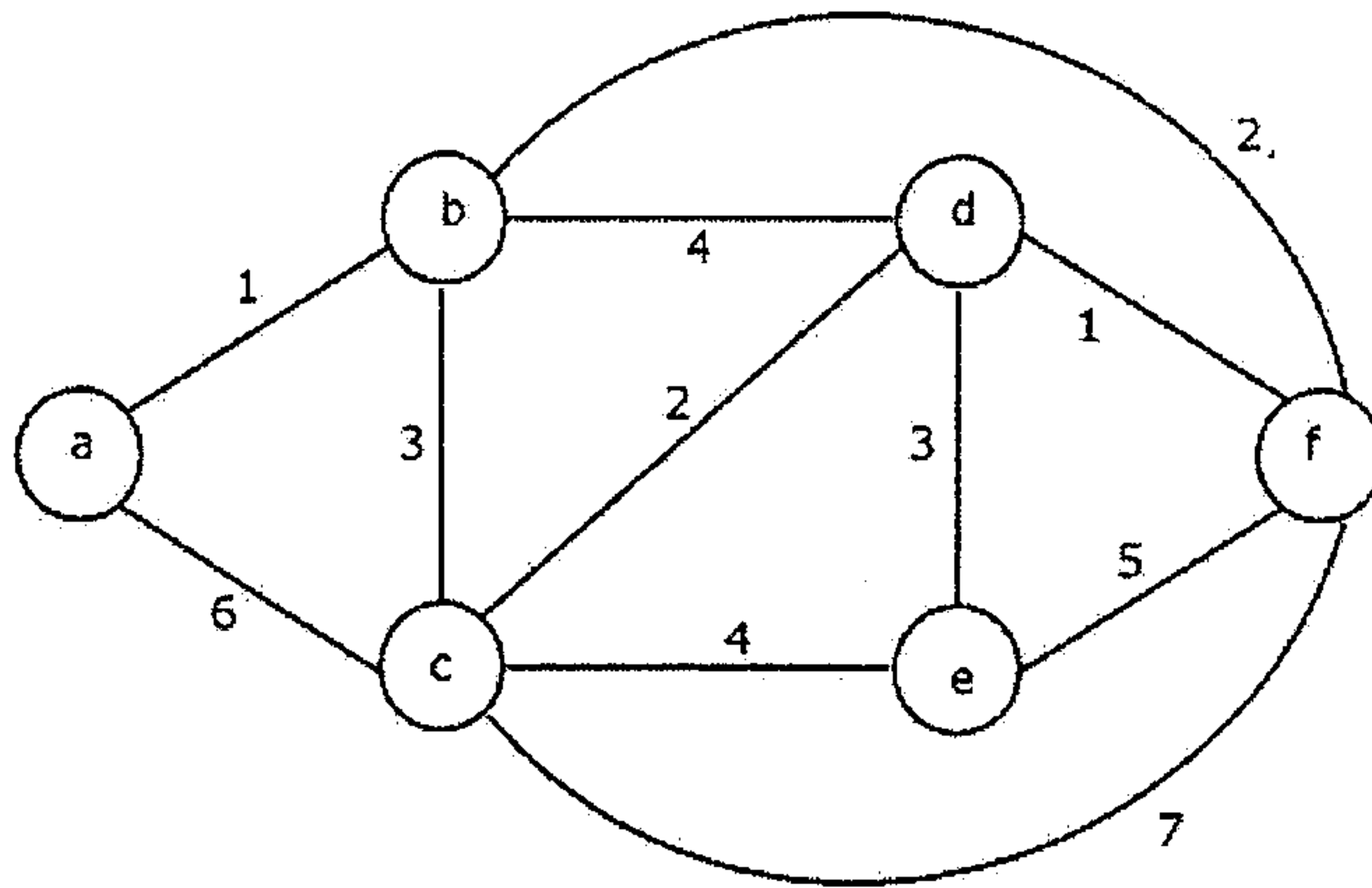


- b) Let A and B be, respectively, the circuit matrix and incidence matrix of a self-loop-free graph G . Prove that $A \times B^T = 0 \pmod{2}$ (4)

- 19 a) Define cut-set matrix and list down any four properties of cut-set matrix (5)

- b) Apply Kruskal's procedure to find the minimum spanning tree from the (5)

following graph G.



- 20 a) Prove that if B is a circuit matrix of a connected graph G with e edges and n vertices, then $\text{rank of } B = e - n + 1$ (5)
- b) How can two linear arrays be used to represent a digraph. Give an example. (5)
Compare this representation with edge list representation in terms of storage.

