Reg No.:_______________ Name:__________________________

APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY
FOURTH SEMESTER B.TECH DEGREE EXAMINATION, DECEMBER 2018

Course Code: MA202
Course Name: PROBABILITY DISTRIBUTIONS, TRANSFORMS AND NUMERICAL METHODS

Max. Marks: 100 Duration: 3 Hours

Normal distribution table is allowed in the examination hall.

PART A (MODULES I AND II)
Answer two full questions.

1 a) Suppose that the probabilities are 0.4, 0.3, 0.2, and 0.1 that there will be 0, 1, 2, or 3 power failures in a certain city during the month of July. Find the mean and variance of this probability distribution. (7)

b) During one stage in the manufacture of integrated circuit chips, a coating must be applied. If 70%of chips receive a thick enough coating. Use Binomial distributionto find the probabilities that, among 15 chips (8)
   (i) at least 12 will have thick enough coating;
   (ii) at most 6 will have thick enough coating;
   (iii) exactly 10 will have thick enough coating.

2 a) If the distribution function of a random variable is given by (7)

$$F(x) = \begin{cases} 1 - \dfrac{1}{x^2} & for \quad x > 1 \\ 0 & for \quad x \le 1 \end{cases}$$

find the probabilities that this random variable will take on a value
   (i) less than 3; (ii) between 4 and 5.

b) In a given city, 6% of all drivers get at least one parking ticket per year. Use the Poisson approximation to the binomial distribution to determine the probabilities that among 80 drivers(randomly chosen in the city): (8)
   (i) 4 will get at least one parking ticket in any given year;
   (ii) at least 3 will get at least one parking ticket in any given year;
   (iii) anywhere from 3 to 6, inclusive, will get at least one parking ticket in any given year.

3 a) Derive mean and variance of uniform distribution. (7)

b) The time required to assemble a piece of machinery is a random variable having approximately a normal distribution with mean 12.9 minutes and standard deviation 2.0 minutes. What are the probabilities that the assembly of a piece of machinery of this kind will take (8)
   (i) at least 11.5 minutes;
   (ii) anywhere from 11.0 to 14.8 minutes?

PART B (MODULES III AND IV)
Answer two full questions.

4 a) Using Fourier cosine integral , show that $\int_0^\infty \frac{\cos xw}{1+w^2} dw = \frac{\pi}{2} e^{-x} \ if \ x > 0.$ (7)

b) Find the Fourier sine transform of $f(x) = \begin{cases} \sin x & if \quad 0 < x < \pi \\ 0 & if \quad x > \pi \end{cases}.$ (8)

5 a) Find the Fourier transform of $f(x) = \begin{Bmatrix} e^{kx} & if \quad x < 0 \\ 0 & if \quad x > 0 \end{Bmatrix}, \; k > 0.$ (7)

b) Find the inverse Laplace transform of $\dfrac{5}{(s^2+1)(s^2+25)}$ using Convolution Theorem. (8)

6 a) Find the Laplace transforms of $(i)\ t\ e^{kt}$ $(ii)\ \cos(wt+\theta)$ (7)

b) Solve the initial value problem $y'' - y' - 6y = 0$, $y(0) = 6$, $y'(0) = 13$ by using Laplace transforms. (8)

PART C (MODULES V AND VI)

Answer two full questions.

7 a) Find the positive solution of $2\sin x = x$ by using Newton-Raphson method, the solution is near to 2. (7)

b) Calculate the Lagrange polynomial $p(x)$ for the 4-D values of the function $f(x)$, $f(1.00) = 1.0000$, $f(1.02) = 0.9888$, $f(1.04) = 0.9784$,and from it find the approximate value of $f(x)$ at $x = 1.005$. (7)

c) Compute $f(1.5)$ from $f(1) = -1, f(2) = -1, f(3) = 1, f(4) = 5$ by using Newton's forward interpolation formula. (6)

8 a) Solve $6x_1 + 2x_2 + 8x_3 = 26,\quad 3x_1 + 5x_2 + 2x_3 = 8,\quad 8x_2 + 2x_3 = -7$ by Gauss Elimination method. (7)

b) Find the value of $(13)^{1/3}$ using Newton Raphson method. (7)

c) Evaluate $\int_0^1 e^{-x^2}\,dx$ by Trapezoidal rule taking 10 subintervals. (6)

9 a) Use Euler's method with $h = 0.1$,compute the value of $y(0.5)$ for the equation $y' = (y+x)^2,\ y(0) = 0.$ (7)

b) Use Runge-Kutta method with $h = 0.1$, compute the value of $y(0.1)$ for the equation $y' = xy^2,\ y(0) = 1.$ (7)

c) Evaluate $\int_0^1 \dfrac{dx}{\cos^2 x}$ by Simpson's rule taking 10 subintervals and compare it with the exact solution. (6)

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