

Reg No.: _____

Name: _____

APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY

Third Semester B.Tech Degree (S,FE) Examination January 2022 (2015 Scheme)

Course Code: CS201**Course Name: DISCRETE COMPUTATIONAL STRUCTURES**

Max. Marks: 100

Duration: 3 Hours

PART A*Answer all questions, each carries 3 marks.*

Marks

- 1 Let $X=\{1,2,3,4\}$ and $R=\{|x>y\}$. Draw the graph of R and give its matrix. (3)
- 2 Assume $A = \{1,2,3\}$ and $\rho(A)$ be its power set. Let \subseteq be the subset relation on power set. Draw the Hasse diagram of $(\rho(A), \subseteq)$. (3)
- 3 Prove that if any 30 people are selected, then we may choose a subset of 5 so that all 5 were born on the same day of the week. (3)
- 4 In how many ways can we arrange "FUZZTONE" so that all vowels come together? (3)

PART B*Answer any two full questions, each carries 9 marks.*

- 5 a) Let $f(x)=x+2$, $g(x)=x-2$, $h(x)=3x$, for $x \in \mathbb{R}$, the set of real numbers. Find $g \circ f$, $f \circ g$, $f \circ h$, $h \circ f$, $h \circ g$ and $f \circ h \circ g$. (4)
- b) Consider a set of integers from 1 to 250. Find (5)
- a) How many of these numbers are divisible by 3 or 5 or 7
- b) How many are divisible by 3 or 7 but not 5.
- c) Number of integers divisible by 3 or 5.
- 6 a) Solve the recurrence relation $a_n = -3a_{n-1} - 3a_{n-2} - a_{n-3}$ with $a_0 = 5$, $a_1 = -9$, and $a_2 = 15$. (5)
- b) Draw the Hasse diagram for divisibility on the set $\{1, 2, 3, 4, 5, 6, 7, 8\}$. (4)
- 7 a) Prove that the set of Idempotent elements of a commutative monoid $\{M, *, e\}$ forms a submonoid of M. (4)
- b) Show that a mapping $f: \mathbb{R} \rightarrow \mathbb{R}$ be defined by $f(x)=ax+b$, where $a, b, x \in \mathbb{R}$, $a \neq 0$ is invertible. Define its inverse. (5)

PART C*Answer all questions, each carries 3 marks.*

- 8 Show that the set $\{0,1,2,3\}$ is not a group under multiplication modulo 4. (3)
- 9 Define ring homomorphism. (3)
- 10 Draw the lattice for $\langle D_{30}, / \rangle$ where D_{30} be the set of all divisors of 30. $/$ denotes the relation divides. (3)
- 11 Explain principle of duality in Boolean algebra. (3)

PART D*Answer any two full questions, each carries 9 marks.*

- 12 a) If $*$ is the operation defined on $S = Q \times Q$, where Q is the set of rational numbers and $*$ is given by $(a,b) * (c,d) = (ac, bc+d)$. Find whether $(S, *)$ is a group? (6)
- b) Let $\langle D_{20}, | \rangle$ denote the poset of all divisors of 20. Show that D_{20} is a lattice. (3)
- 13 a) Explain Distributive lattice with an example. (4)
- b) Show that (Z, θ, \oplus) is a ring where $a \theta b = a+b-1$ and $a \oplus b = a+b-ab$ (5)
- 14 a) Prove that the order of a subgroup of a finite group divides the order of the group. (4)
- b) Simplify the boolean algebraic expression $AB+A(B+C)+B(B+C)$. (5)

PART E*Answer any four full questions, each carries 10 marks.*

- 15 a) Show that $(t \wedge s)$ can be derived from premises $p \rightarrow q, q \rightarrow \neg r, r, p \vee (t \wedge s)$. (5)
- b) Symbolize the following statement. (i). All men are giants. (ii). Given any positive integer there is a greater positive integer. (5)
- 16 a) Show that the following premises are in consistent. (5)
If Ram gets his degree he will go for a job. If he goes for a job he will get married soon. If he goes for higher study he will not get married. Ram gets his degree and he goes for higher study.
- b) Prove by contra positive that if n^2 is even integer then n is even. (5)
- 17 a) Show that $(a \rightarrow b) \wedge (a \rightarrow c), \neg (b \wedge c), (d \vee a) \Rightarrow d$ (5)
- b) Show that from $(\exists x)(F(x) \wedge S(x)) \rightarrow \forall y(M(y) \rightarrow W(y))$
 $(\exists y)(M(y) \wedge \neg W(y))$
Concludes $(x) (F(x) \rightarrow \exists x S(x))$.

- 18 a) Construct truth table for (5)
- (i) $(p \leftrightarrow q) \leftrightarrow ((p \wedge q) \vee (\neg p \wedge \neg q))$
- b) Show that premises “All men are mortal“ and Socrates is a man “implies “ (5)
Socrates is a Mortal“.
- 19 a) Show that $(p \rightarrow q) \wedge (q \rightarrow r) \rightarrow (p \rightarrow r)$ is a tautology. (5)
- b) Prove by mathematical induction that $6^{(n+2)} + 7^{(2n+1)}$ is divisible by 43 for each (5)
positive integer n.
- 20 a) Use a truth table to verify the distributive law $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$. (5)
- b) Prove that $23^n - 1$ is divisible by 11 for all positive integers n. (5)
