Reg No.:_______________ Name:__________________________

APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY

THIRD SEMESTER B.TECH DEGREE EXAMINATION, DECEMBER 2018

Course Code: CS201

Course Name: DISCRETE COMPUTATIONAL STRUCTURES

Max. Marks: 100 Duration: 3 Hours

PART A

Answer all questions, each carries 3 marks. Marks

1		What non zero entries are there in the relation matrix of intersection of R and its converse if R is an anti-symmetric relation?	(3)
2		Define a recurrence relation. Give an example	(3)
3		Show that the set of idempotent elements of any commutative monoid forms a submonoid.	(3)
4		Let f: R → R be given by $f(x) = x^3 – 2$, where R is the set of real numbers. Find f^{-1}	(3)

PART B

Answer any two full questions, each carries 9 marks.

5	a)	Draw the Hasse diagram for the following sets under the partial ordering relation “Divides”, and indicates those which are totally ordered. {2,6,24}, {1,2,3,6,12}, {2,4,8,16}, {3,9,27,54}	(5)
	b)	Prove that every equivalence relation on a set generates a unique partition of the set with the blocks as R-equivalence classes.	(4)
6	a)	Solve the recurrence relation T(k) – 7 T(k-1) + 10 T(k-2) = 6 + 8k with T(0) = 1 and T(1) = 2.	(5)
	b)	What is the minimum number of students required in anEnglish class to be surethat at least six will receive the same grade, if there are five possible grades?	(4)
7	a)	From a group of 7 men and 6 women, 5 people are to be selected to form a committee, such that at least 3 men are there in the committee. In how many ways can the committee be formed?	(4)
	b)	Let f: R → R and g: R → R, where R is the set of real numbers. Find $f \circ g$ and $g \circ f$, where $f(x) = x^2 – 2$ and $g(x) = x + 4$. State whether these functions are injective, surjective, and bijective.	(5)

PART C

Answer all questions, each carries 3 marks.

8		How many proper subgroups will be there for a group of order 11? Justify your answer.	(3)
9		Show that every chain is a distributive lattice.	(3)
10		What conditions to be satisfied if an algebraic system (A, +, .) is called a ring?	(3)
11		Define a complemented lattice. Give an example	(3)

PART D

Answer any two full questions, each carries 9 marks.

12	a)	For a cyclic group of order n generated by an element a, show that n is the least	(5)

positive integer for which $a^n = e$. (e is the identity element)

b) Show that every finite integral domain is a field. (4)

13 a) Let (L, *, $\oplus$) be any distributive lattice. For any a, b, c element of L, show that (a*b = a*c) and (a $\oplus$ b = a $\oplus$ c) ==> b = c. (4)

b) Define a Boolean algebra. Illustrate a two element Boolean Algebra with an example (5)

14 a) Prove that every finite group of order n is isomorphic to a permutation group of degree n. (5)

b) Define Lattice homomorphism and direct product of two lattices. (4)

PART E

Answer any four full questions, each carries 10 marks.

15 a) Without using truth tables prove that $\urcorner(P \wedge Q) \rightarrow (\urcorner P \vee (\urcorner P \vee Q)) \Leftrightarrow (\urcorner P \vee Q)$ (5)

b) Using truth table determine whether the conclusion $\urcorner P$ follows logically from the premises $P \rightarrow Q$ and $\urcorner(P \wedge Q)$. (5)

16 a) Show that $R \vee S$ follows logically from the premises $C \vee D$, $(C \vee D) \rightarrow \urcorner H$, $\urcorner H \rightarrow (A \wedge \urcorner B)$, and $(A \wedge \urcorner B) \rightarrow (R \vee S)$. (4)

b) If there was a ball game, then travelling was difficult. If they arrived on time, then travelling was not difficult. They arrived on time. Therefore, there was no ball game. Show that these statements constitute a valid argument. (6)

17 a) Symbolise the statement " x is the father of the mother of y" (4)

b) Indicate the variables that are free and bound (6)

i. $(x)\,(P(x) \wedge R(x)) \rightarrow (x)P(x) \wedge R(x)$

ii. $(x)(P(x) \wedge (\exists x)\,Q(x)) \vee ((x)P(x) \rightarrow Q(x))$

18 a) Use mathematical induction to prove that $n^3 + 2n$ is a multiple of 3 for all $n \geq 1$. (5)

b) Show that (5)

$(\exists x)(P(x) \wedge Q(x)) \Rightarrow (\exists x)P(x) \wedge (\exists x)Q(x)$

19 a) Test the consistency of the following statements. (5)

"If Jack misses many shootings, then his films will be a flop. If Jack's film flops, then he fails to act. If he signs for many films, then he does not fail to act." Jack misses many shootings and signs for many films.

b) Show that $R \rightarrow S$ can be derived from $P \rightarrow (Q \rightarrow S)$, $R \vee P$ and Q (5)

20 a) Using proof by contrapositive method, prove that "if 3n + 2 is odd, then n is odd" (5)

b) Use mathematical induction to prove that $1^2 + 2^2 + 3^2 + \ldots + n^2 = (n/6)(n+1)(2n+1)$ for all $n \in N$. (5)

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