Reg No.:_____   Name:_____

APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY
THIRD SEMESTER B.TECH DEGREE EXAMINATION, APRIL 2018

Course Code: CS201

Course Name: DISCRETE COMPUTATIONAL STRUCTURES (CS, IT)

Max. Marks: 100   Duration: 3 Hours

PART A
Answer all questions, each carries 3 marks   Marks

1. Let X = {1,2,3,4} and R = {<x,y>| x > y} . Draw the graph of R and also give its matrix. (3)

2. Define countable and uncountable set. Prove that set of real numbers are uncountable. (3)

3. State Pigeonhole principle. A school has 550 students. Show that at least two of them were born on the same day of the year. (3)

4. How many 4-digit numbers can be formed from six digits 1, 2, 3, 5, 7, 8. Also find how many numbers are less than 4500. (3)

PART B
Answer any two full questions, each carries 9 marks

5. a) Let Z be the set of integers and R be the relation called congruence modulo 3 defined by R = {<x,y>| x and y are elements in Z and (x-y) is divisible by 3}. Determine the equivalence classes generated by the elements of Z. (5)

   b) Let A be the set of factors of a particular positive integer m and let <= be the relation divides, ie relation <= be such that x<=y if x divides y. Draw the Hasse diagrams for m= 30 and m= 45. (4)

6. a) Let f(x) = x+2, g(x) = x-2 and h(x) =3x for x is in R, where R is the set of real numbers. Find gof, fog, (foh)og , hog . (4)

   b) Among 100 students, 32 study mathematics, 20 study physics, 45 study biology, 15 study mathematics and biology, 7 study mathematics and physics, 10 study physics and biology and 30 do not study any of the three subjects. (5)
   i) Find the number of students studying all three subjects.
   ii) Find the number of students studying exactly one of three subjects.

7. a) Solve the recurrence equation $a_r + 5\,a_{r-1} + 6\,a_{r-2} = 42\ 4^r$ where $a_2 = 278$ and $a_3 = 962$. (4)

   b) Define Monoid. Show that the algebraic systems $<Z_m , +_m>$ and $<Z_m , *_m>$ are monoids where m = 6. (5)

PART C
Answer all questions, each carries 3 marks

8. Define Abelian group. Prove that the algebraic structure $<Q^+, *>$ is an abelian group. * defined on Q^+ by a * b = (ab)/2. (3)

9. Define Cosets and Lagrange's theorem. (3)

10. Draw the diagram of lattices $<S_n, D>$ for n = 15 and n = 45. Where S_n be the set (3)

of all divisors of n and D denote the relation 'divides'.

11   Define sub Boolean algebra. Give one example.   (3)

PART D
Answer any two full questions, each carries 9 marks

12   a)   Show that the set{ 0, 1, 2, 3, 4, 5 } under addition and multiplication modulo 6 is   (5)
           group or not.
     b)   Find all the subgroups of $< Z_{12}, +_{12} >$ (4)
13   a)   Define ring and field. Give one example to each.   (5)

     b)   A= {2, 3, 4, 6, 12, 18, 24, 36} with partial order of divisibility. Determine   (4)
           whether the POSET is a lattice or not.
14   a)   Show that the lattice < Sn, D > for n = 216 is isomorphic to the direct product of   (5)
           lattices n = 8 and n = 27.
     b)   Define complemented lattice and distributive lattice. Give one example to each.   (4)

PART E
Answer any four full questions, each carries 10 marks

15   a)   Prove that p v (q ^ r) and (p v q)^(p v r) are logically equivalent   (5)
     b)   Show that (~p ^ (~q ^ r)) v (q ^ r) v (p ^ r) <=> r   (5)
16   a)   Show that s v r is tautologically implied by (p v q) ^ (p -> r) ^ (q -> s)   (5)
     b)   Show that r ^ (p v q) is a valid conclusion from the premises p v q, q -> r,   (5)
           p -> m, and ~m
17   a)   "If there are meeting, then traveling was difficult. If they arrived on time, then   (6)
           traveling was not difficult. They arrived on time. There was no meeting". Show
           that the statements constitute a valid argument.
     b)   Construct truth table for ~ (p ^ q) <-> (~p v ~q). Determine whether it is   (4)
           tautology or not.
18   a)   Show that (x) (P(x) -> Q(x)) ^ (x) (Q(x) -> R(x)) => (x) (P(x) -> R (x))   (5)
     b)   Prove that (∃x) (P(x) ^ Q(x)) => (∃x) P(x) ^ (∃x) Q(x)   (5)
19   a)   Symbolize the statements: (4)
           i) All the world loves a loverii) All men are giants.
     b)   Show that (∃x) M(x) follows logically from the premises (x) (H(x) -> M(x)) and   (6)
           (∃x) H(x)
20   a)   Prove by contradiction that if n^2 is an even integer then n is even.   (5)
     b)   Prove that $23^n - 1$ is divisible by 11 for all positive integers n.   (5)

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