

Reg No.: _____

Name: _____

APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY
FIFTH SEMESTER B.TECH DEGREE EXAMINATION, DECEMBER 2017

Course Code: CS309

Course Name: GRAPH THEORY AND COMBINATORICS (CS)

Max. Marks: 100

Duration: 3 Hours

PART A

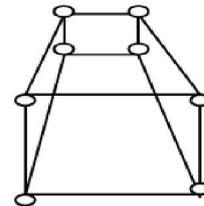
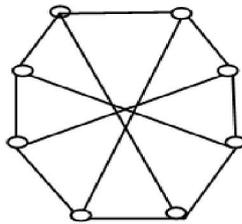
Answer all questions, each carries 3 marks.

- | | | Marks |
|---|--|-------|
| 1 | Consider a graph G with 4 vertices: v_1, v_2, v_3 and v_4 and the degrees of vertices are 3, 5, 2 and 1 respectively. Is it possible to construct such a graph G? If not, why? | (3) |
| 2 | Draw a disconnected simple graph G1 with 10 vertices and 4 components and also calculate the maximum number of edges possible in G1. | (3) |
| 3 | State Dirac's theorem for hamiltonicity and why it is not a necessary condition for a simple graph to have a Hamiltonian circuit. | (3) |
| 4 | Differentiate between symmetric and asymmetric digraphs with examples and draw a complete symmetric digraph of four vertices. | (3) |

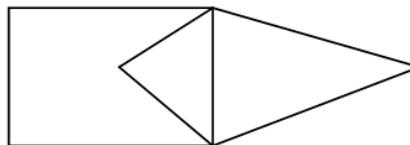
PART B

Answer any two full questions, each carries 9 marks.

- 5 a) What are the basic conditions to be satisfied for two graphs to be isomorphic? Are the two graphs below isomorphic? Explain with valid reasons (6)



- b) Write any two applications of graphs with sufficient explanation (3)
- 6 a) Consider the graph G given below: (4)



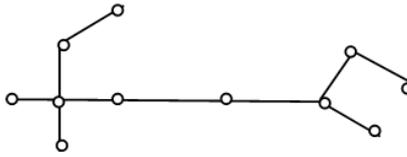
- Define Euler graph. Is G an Euler? If yes, write an Euler line from G.
- b) What is the necessary and sufficient condition for a graph to be Euler? And also prove it. (5)
- 7 a) Define Hamiltonian circuits and paths with examples. Find out the number of edge-disjoint Hamiltonian circuits possible in a complete graph with five vertices (5)
- b) State Travelling-Salesman Problem and how TSP solution is related with Hamiltonian Circuits? (4)

PART C

Answer all questions, each carries 3 marks.

- 8 List down any two properties of trees and also prove the theorem: *A graph is a tree if and only if it is a minimally connected.* (3)

- 9 Consider the tree T, given below (3)



- 10 Label the vertices of T appropriately and find the center and diameter of T. Prove the statement: (3)

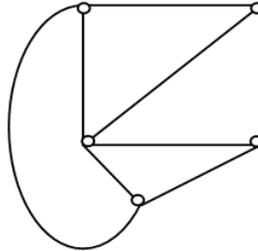
Every cut-set in a connected graph G must also contain at least one branch of every spanning tree of G

- 11 List down the properties stating the relationship between the edges of graph G and its dual G^* (3)

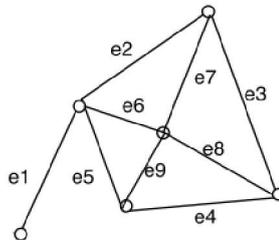
PART D

Answer any two full questions, each carries 9 marks.

- 12 a) Define spanning trees. Consider the graph G given below and obtain any *three* spanning trees from G. Calculate the number of distinct spanning trees possible from a complete graph with n vertices. (5)

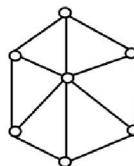


- b) Let $G = (V, E)$ be a connected graph, and let $T = (V, S)$ be a spanning tree of G. Let $e = (a, b)$ be an edge of G *not in* T. Prove that, for any edge f on the path from a to b in T, $(V, (S \cup \{e\}) - \{f\})$ is another spanning tree for G (4)
- 13 a) Define cut set. Find any four cut sets from the graph G given below and also find the edge connectivity of G. (5)



- b) Define vertex connectivity and draw a graph with an articulation point. (3)
- c) State Euler's Theorem (*formula*). (1)

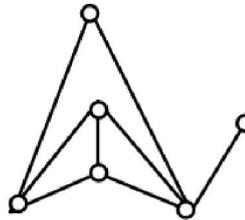
- 14 a) Draw two Kuratowski's graphs and also prove that Kuratowski's first graph is non planar using appropriate inequality. (4)
- b) Draw the geometric dual (G^*) of the graph G given below and also check whether G and G^* are self dual or not, substantiate your answer clearly? (5)



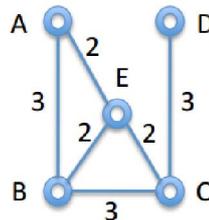
PART E

Answer any four full questions, each carries 10 marks.

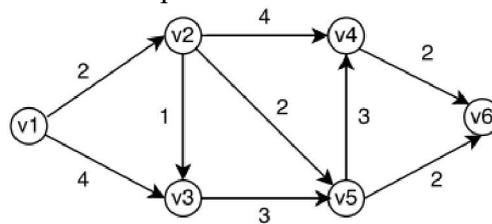
- 15 a) List down any four properties of adjacency matrix (4)
- b) Construct an adjacency matrix(X) for the following graph and also mention how the concept of edge sequences is described with X^3 (no need to find X^3 from X) (6)



- 16 a) Prove the theorem: (4)
If $A(G)$ is an incidence matrix of a connected graph G with n vertices, the rank of $A(G)$ is $n-1$
- b) Describe with examples the usage of incidence matrix to find two graphs (g_1 and g_2) are isomorphic. (6)
- 17 a) Define cut-set matrix with an example and list down any four properties of cut-set matrix (6)
- b) If B is a circuit matrix of a connected graph G with e edges and n vertices, then show that the number of linearly independent rows in $B = e-n+1$ (4)
- 18 a) Draw the flow chart of minimum spanning-tree algorithm. (7)
- b) Find MST from the graph given below by simply applying Kruskal's procedure. (3)



- 19 Write the Dijkstra's shortest path algorithm (no need to draw flowchart). Apply this algorithm to find the shortest path between v_1 and v_6 (10)



- 20 Draw the flowchart of *Connectedness and Components* algorithm and also apply this algorithm on any graph (G) with 2 components. (10)
